Preparatory Exercises for Integrals

SS2020 - Analysis 2 - University of Leipzig Mahsa Sayyary Namin

Problem 1. Find the anti-derivatives of the following functions in their domain.

(1) $xe^x + \sin x$

(2) $x.2^x$

(3) lnx

(4) $tan^{-1}x$

(5) $\frac{1}{x^2-5x+6}$

Problem 2. Compute the following integrals.

(6) $\int_{-3}^{6} |t^2 - 4| dt$

(7*) $\int_0^{\pi} \cos(99\theta)\sin(101\theta)d\theta$

(8) $\int_0^{\frac{\pi}{4}} \sec^2(y)\sqrt{2 + tan(y)}dy$

Problem 3. Without integrating, determine whether the following integral exists or not.

(9) $\int_1^{\infty} \frac{dx}{\sqrt{x+1}}$

$\star$ Compare it to the other easier integrals and use convergence/divergence of the sequences that you know.

Problem 4. Compute the following indefinite integrals. (i.e., find an anti-derivative for each of the functions in front of $\int$)

(10) $\int \frac{\cos x}{\sin x(1-\sin x)}\, dx$

(11) $\int \frac{2-x}{x^2+1}\, dx$

(12) $\int \sin(\ln x) dx$

(13*) $\int \frac{\tan^3(\ln x)}{x}\, dx$

Problem 5. Decide that which of the following integrals exists and which does not. Explain your answer and find the existing ones.

(14) $\int_2^4 \frac{dx}{(x-3)^3}$

(15) $\int_1^\infty \frac{1}{x \ln x}\, dx$

(16) $\int_0^\infty \frac{1}{4+x^2}\, dx$

Problem 6. Find the area bounded by the x-axis and the given curve:

(17) $4 \sin x \cos^3 x \qquad x \in [0, \frac{\pi}{2}]$

Problem 7. Use integral to compute the length of the curve $C = im(\gamma)$, where $\gamma : [0, 2\pi] \to \mathbb{R}^2$ is defined by $\gamma(t) = (\sin t, \cos t)$. Does that confirm your previous knowledge on the length of the unit circle?

Problem 8. What is the length of the curve $y = \frac{1}{2}x^2$ for $x \in [0, 1]$.

Good Luck.