Problem Sheet 15

Due Date: 20.07.2020, 12:00 UTC+2 (CEST)

Bonus! each point will be *doubled: Each problem is worth $x+\ x^*$ points, where x is the amount in [] below. So the maximum that counts to your exam admission percentage of this sheet is 21, but you can obtain even 42 points!

Problem 1. [**3** pts] Compute the directional derivative of the function $F = (F_1, F_2)$, where

$$F_1(x,y,z) = sin(x+y) - e^z - x, \quad F_1(x,y,z) = \ln z - 5x^z$$

$(1,-1,1)$ in the direction $v = (1,0,3)$

Problem 2. [**5** pts] Find the Frenet frame of the curve C given by the following parametrisation

$$\gamma(t) = (sint, cost, 2t)$$

(warning: is the above the arc-length parametrisation?)

Problem 3. [**3** pts]
Find the tangent and normal spaces to the ellipsoid:

$$\left\{(x,y,z) \mid \frac{x^2}{3} + \frac{y^2}{2} + \frac{z^2}{6} = 5\right\}$$

at the point $(1,1,5)$.

Problem 4. [**3+3** pts] Find local extrema of the following functions:

- $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x,y) = (x+y)e^{-(x^2+y^2)}$
- $f : [0,1]^2 \to \mathbb{R}, \quad f(x,y) = x^3 + y^3 - 3xy$

Problem 5. [**4** pts] Consider

$$(x^2 + y^2 + z^4)^{\frac{1}{2}} - \cos y - \cos z = 0.$$

Is y a dependent variable around the point $(2,0,0)$? If yes, compute the differential of the implicit function at $(x_0, z_0) = (2,0)$.
Is x a dependent variable around the point $(2,0,0)$? If yes, compute the differential of the implicit function at $(y_0, z_0) = (0,0)$.