HOW TO FIND THE TRANSITION FUNCTIONS IN 3.C

An easy way is to start with a sufficiently big example and you will see how the general formula has been made. We have the maps $\phi_i : \mathbb{C}^n \to U_i$ defined by

$$\phi_0(z_1, \dots, z_n) = [1, z_1 : \dots : z_n]$$
$$\phi_i(z_1, \dots, z_n) = [z_1 : \dots : z_i : 1 : z_{i+1} : \dots : z_n] \qquad 1 \leq i \leq n$$

where $U_i = \{[z_0 : \dots : z_n] | z_i \neq 0\}$. You can find the map $\phi_i^{-1} : U_i \to \mathbb{C}^n$ that is:

$$\phi_0^{-1}([z_0 : \dots : z_n]) = (\frac{z_1}{z_0}, \dots, \frac{z_n}{z_0})$$
$$\phi_i^{-1}([z_0 : \dots : z_n]) = (\frac{z_0}{z_i}, \dots, \frac{z_{i-1}}{z_i}, \frac{z_{i+1}}{z_i}, \dots, \frac{z_n}{z_i}) \qquad 1 \leq i \leq n.$$

Let $n = 8$, $j = 4$ and then set i to be 1,3,5, and 7:

$$\phi_i^{-1}(\phi_4(z_1, \dots, z_8)) = \phi_i^{-1}([z_1 : z_2 : z_3 : z_4 : 1 : z_5 : z_6 : z_7 : z_8]),$$

We have:

$$\phi_1^{-1}([z_1 : z_2 : z_3 : z_4 : 1 : z_5 : z_6 : z_7 : z_8]) = (\frac{z_1}{z_2}, \frac{z_3}{z_2}, \frac{z_4}{z_2}, \frac{1}{z_2}, \frac{z_5}{z_2}, \frac{z_6}{z_2}, \frac{z_7}{z_2}, \frac{z_8}{z_2}),$$
$$\phi_3^{-1}([z_1 : z_2 : z_3 : z_4 : 1 : z_5 : z_6 : z_7 : z_8]) = (\frac{z_1}{z_4}, \frac{z_2}{z_4}, \frac{z_3}{z_4}, \frac{1}{z_4}, \frac{z_5}{z_4}, \frac{z_6}{z_4}, \frac{z_7}{z_4}, \frac{z_8}{z_4}),$$
$$\phi_5^{-1}([z_1 : z_2 : z_3 : z_4 : 1 : z_5 : z_6 : z_7 : z_8]) = (\frac{z_1}{z_5}, \frac{z_2}{z_5}, \frac{z_3}{z_5}, \frac{z_4}{z_5}, \frac{1}{z_5}, \frac{z_6}{z_5}, \frac{z_7}{z_5}, \frac{z_8}{z_5}),$$
$$\phi_7^{-1}([z_1 : z_2 : z_3 : z_4 : 1 : z_5 : z_6 : z_7 : z_8]) = (\frac{z_1}{z_7}, \frac{z_2}{z_7}, \frac{z_3}{z_7}, \frac{z_4}{z_7}, \frac{1}{z_7}, \frac{z_5}{z_7}, \frac{z_6}{z_7}, \frac{z_8}{z_7}).$$

Please also add the cases where i or j are 0 to the main formula.

I hope that it helps. :)

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