UNIVERSITÄT LEIPZIG

Problem Sheet 9

Due Date: 27.04.2020, 12:00 UTC+2 (CEST)

Problem 1. [**3** pts] Solve the following system of linear of equations.

$$\begin{cases} 3x_1 + 9x_2 - 2x_3 + 17x_4 - 13x_5 = 16 \\ 2x_1 + 7x_2 + 7x_4 - 2x_5 = 11 \\ 2x_1 + 5x_2 - 2x_3 + 13x_4 - 13x_5 = 11 \\ x_1 + 3x_2 - x_3 + 5x_4 - 4x_5 = 5 \end{cases} \tag{1}$$

Problem 2. [**3** pts] Which values of $r \in \mathbb{R}$ make the following system solvable? Find these solutions.

$$\begin{cases} x_1 + 2x_2 + 2x_3 + x_4 = 3 \\ 3x_1 + 8x_2 + 8x_3 + 7x_4 = 9 \\ 2x_1 + 5x_2 + 6x_3 + 5x_4 = 7 \\ rx_4 + x_1 + 3x_2 + 4x_3 = 5 \end{cases} \tag{2}$$

Problem 3. [**2+2+1+3*** pts] Compute determinants of

i) $A = \begin{bmatrix} 3 & 4 & 2 & 2 \\ 4 & 5 & 6 & 5 \\ 2 & 3 & 6 & 0 \\ 8 & 7 & 7 & 8 \end{bmatrix}$

ii) $B = \begin{bmatrix} 3 & 6 & 0 & 6 & 3 \\ 4 & 5 & 0 & 4 & 2 \\ 5 & 4 & 3 & 3 & 2 \\ 4 & 3 & 1 & 2 & 2 \\ 1 & 2 & 1 & 2 & 1 \end{bmatrix}$

iii) $det(A^5 B^2)$

iv) (*) show that for the following $n \times n$ matrix

$$det \begin{bmatrix} 1 & 1 & ... & 1 \\ a_1 & a_2 & ... & a_n \\ a_1^2 & a_2^2 & ... & a_n^2 \\ . & & & \\ . & & & \\ . & & & \\ a_1^{n-1} & a_2^{n-1} & ... & a_n^{n-1} \end{bmatrix} = \Pi_{j>i}(a_j - a_i)$$

Problem 4. [**3+2** pts] Show that A^{-1} of a triangular matrix A is a triangular matrix. What can you say about relation between $[A]_{ii}$ and $[A^{-1}]_{ii}$? (diagonals of A and A^{-1})

Problem 5. [**2+2+2** pts] Invert matrices

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 3 & 2 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & -1 & -2 & 2 \\ -1 & -2 & 2 & -1 \\ 3 & 1 & -1 & 2 \end{bmatrix}$$

and solve (if possible) $A_i x = b_i$, $i = 1, 2$ with A_i above and $b_1 = \begin{bmatrix} 3 \\ 5 \\ 0 \\ 6 \end{bmatrix}$, $b_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Problem 6. [**3** pts] What are the values of parameters $s, t \in \mathbb{R}$ such that

$$c_1 = (5, 7, s, 2), c_2 = (1, 3, 2, 1), c_3 = (2, 2, 4, t)$$

of $\mathbb{R}^4$ are linearly independent?

Problem 7. [**3** pts] Gram-Schmidt orthonormalize
$\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ (starting with the last one)

Problem 8. [**2+3+1** pts] We denote

a) (e_i) - the canonical basis

b) ϵ_{ijk} - alternating symbol (Levi-Civita symbol)

c) δ_{ij} - Kronecker delta

Prove the following $\mathbb{R}^3$ identities

i) the $\epsilon - \delta$ identity : $\epsilon_{ijk}\epsilon_{ipq} = \delta_{jp}\delta_{kq} - \delta_{jq}\delta_{kp}$

ii) $u \times (v \times w) = (u \cdot w)v - (u \cdot v)w$

iii) $u \times (v \times w) + v \times (w \times u) + w \times (u \times v) = 0$