Problem Sheet 14
Due Date: 13.07.2020, 12:00 UTC+2 (CEST)

Problem 1. [**3** pts] Compute the differential and the Jacobian determinant of

$$f(r, \theta, \phi) = (r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta), \qquad \text{for } r > 0, \ 0 \le \theta < \pi, \ 0 \le \phi < 2\pi$$

Problem 2. [**2** pts] Let $f : \mathbb{R}^2 \to \mathbb{R}$ be diffetentiable. Find the differential of

$$g(x, y, z) := f(x + y + z, x^2 + y^2 + z^2)$$

Problem 3. [**3** pts]
Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$f(x, y) = (2xy, e^x + y)$$

Show that f is invertible in a neighbourhood of $(1, 1)$.

Problem 4. [**3** pts] Consider $f : \mathbb{R}^d \to \mathbb{R}^d$ given by $f(x) = x|x|$. Discuss its differentiability.

Problem 5. [**3** pts] Consider a differentiable function $f : \mathbb{R} \to \mathbb{R}^3$. Show that

$$|f(x)| = 1 \implies f \cdot \nabla f = 0$$